2025 WAMC 12 Problems

1. What is the sum of the digits of

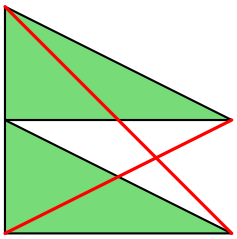
$$\frac{202503 + 202505 + 202512 + 202513 + 202517}{5}?$$

(A) 7 **(B)** 9 **(C)** 10 **(D)** 14 **(E)** 19

2. Points A, B, and C lie on circle ω with center O. If $\angle OAB = \angle OAC = 20°$, find $\angle OBC$.

(A) $10°$ **(B)** $20°$ **(C)** $40°$ **(D)** $50°$ **(E)** $70°$

3. In the figure below, the sum of the squares of the lengths of the red segments is 10. The two congruent light blue right triangles have leg lengths in the ratio $1 : 2$. Find their total area.



(A) $\frac{6}{7}$ **(B)** $\frac{15}{13}$ **(C)** $\frac{9}{7}$ **(D)** $\frac{10}{7}$ **(E)** $\frac{20}{13}$

4. Arjun, Vincent, Jacob, and Alexander stand in a line. Arjun refuses to stand in the first or second position. Vincent refuses to stand in the second or third position. Jacob refuses to stand in the third or fourth position. Alexander refuses to stand in the fourth or first position. How many possible orderings are there?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

5. Ryan writes down the number 11111122. Patrick randomly rearranges the digits into a new eight-digit number. What is the probability that the sum of the digits in the left half equals the sum of the digits in the right half?

(A) $\frac{1}{4}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{5}{7}$

6. Bruce rolls a die numbered 1 through 10. Each face of the die is weighted so that the probability of rolling n is directly proportional to n. What is the expected value of one spin?

(A) $\frac{11}{2}$ **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

7. In a math competition with 100 participants ranked from 1 to 100, Ryan randomly chooses an integer $N > 4$, then randomly interviews 2 of the top N participants. What is the probability that both the first and second-ranked participants are interviewed?

(A) $\frac{1}{2500}$ **(B)** $\frac{1}{1000}$ **(C)** $\frac{1}{500}$ **(D)** $\frac{1}{200}$ **(E)** $\frac{1}{100}$

8. Let p be a prime for which there exist positive integers a and b satisfying $\sqrt{a^2+p}=b^2$. Find the sum of the four smallest possible values of p.

(A) 104 **(B)** 116 **(C)** 126 **(D)** 138 **(E)** 144

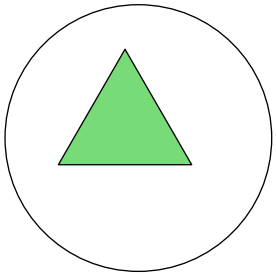
9. For how many integers $1 \leq n \leq 10^{100}$ is

$$\log_2 \log_5 n - \log_2 \log_5 10$$

an integer?

(A) 0 **(B)** 1 **(C)** 3 **(D)** 6 **(E)** 7

10. Laura takes a sponge in the shape of an equilateral triangle with side length 2 and dips it in paint. Then, without rotating the sponge or painting outside the circle, she paints as much of the interior of a circle with radius 2 as she can. How much of the circle is left unpainted?



(A) $2\pi - 3\sqrt{3}$ **(B)** $2\pi - 3\sqrt{2}$ **(C)** $2\pi - 2\sqrt{3}$ **(D)** $4\pi - 3\sqrt{3}$ **(E)** $4\pi - 2\sqrt{3}$

11. Ryan places a standard fair 6-sided die (with opposite faces summing to 7) on the table with the 1 face on top. He then rolls the die to an adjacent face three times in succession. What is the probability that no number greater than 3 ever appears on the top?

(A) $\frac{1}{216}$ **(B)** $\frac{1}{64}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{1}{6}$

12. What is the area formed by all coordinate pairs (x, y) that satisfy

$$(x-x_1)^2 + (y-y_1)^2 \leq 4$$

over all x_1 and y_1 that satisfy $x_1^2 + y_1^2 = 200$?

(A) $80\pi\sqrt{2}$ **(B)** $100\pi\sqrt{2}$ **(C)** 200π **(D)** 204π **(E)** $160\pi\sqrt{2}$

13. Triangle $\triangle ABC$ intersects circle ω at points D, E, F, G, H, I such that $ADEB$, $BFGC$, and $CHIA$ are collinear in those orders. Given that $AD = 3$, $AI = 4$, $HC = 4$, $GC = 5$, and $DE + FG = 8$, find the value of DE.

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

14. How many ways are there to rearrange the letters in the word NAGGING so that no G's are adjacent?

(A) 48 **(B)** 60 **(C)** 120 **(D)** 168 **(E)** 240

15. The solutions to the equation $x^6+x^5+x^4+2x^3+x^2+x+1=0$ can be expressed as $e^{2i\pi a_k}$, where $1 \leq k \leq 6$ and $0 < a_k < 2\pi$. Compute $a_1 + a_2 + a_3 + a_4 + a_5 + a_6$.

(A) 0 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 12

16. Real numbers x and y satisfy the system of equations

$$\begin{aligned} x^3 - 8x^2 + px - q &= 0, \\ y^3 - 8y^2 + py - q &= 0, \text{ and} \\ (x+y)^3 - 8(x+y)^2 + p(x+y) - q &= 0. \end{aligned}$$

There exists a unique k for which $kp + q$ is constant for all values of x and y. Find the value of $kp + q$.

(A) -64 **(B)** -58 **(C)** -32 **(D)** -10 **(E)** 26

17. Chords AB and AC of a circle with center O have lengths 10 and 16 and midpoints M and N, respectively. Line MN is extended to meet the circle at X and Y, so that X and N are on opposite sides of AB. Let Z be the reflection of X over M. If $\angle MON = 120^\circ$, compute YZ.

(A) $\frac{44}{7}$ **(B)** $\frac{88}{7}$ **(C)** 14 **(D)** 20 **(E)** $\frac{160}{7}$

18. Four consecutive natural numbers a, b, c, d satisfy $a < b < c < d$. If a is divisible by 3, b is divisible by 5, c is divisible by 7, and d is divisible by 9, find the minimum value of $a + b + c + d$.

(A) 82 **(B)** 106 **(C)** 510 **(D)** 642 **(E)** 654

19. Isosceles trapezoid $ABCD$ has $AB \parallel CD$ and $AB = 3$. Triangle ABE is drawn outside the trapezoid with $\angle ABE = 90^\circ$. If $EC = 25$, $ED = 26$, and the area of pentagon $AEBCD$ is 104, compute the area of triangle BDE.

(A) 54 **(B)** 76 **(C)** 80 **(D)** 104 **(E)** 204

20. For how many integers n between 1 and 100, inclusive, is $\frac{n!^5}{5^n}$ an integer?

(A) 25 **(B)** 30 **(C)** 50 **(D)** 75 **(E)** 80

21. Triangle $\triangle ABC$ has $AB = 11$, $AC = 2$, and circumcircle ω. Let D be a point on side AB, and point E is a point on arc BC of ω not including A satisfying $DE \perp BC$. Another point F on ω satisfies $\angle CBF = 90^\circ$. Given that $BE = BF\sqrt{3} = EF\sqrt{3}$, find BD.

(A) $\frac{41}{8}$ **(B)** $\frac{49}{8}$ **(C)** $\frac{49\sqrt{3}}{12}$ **(D)** $\frac{75}{8}$ **(E)** $7\sqrt{3}$

22. How many ordered pairs (a, b) are there to the equation

$$\gcd(a,b)^3 + 90\gcd(a,b) = (\gcd(a,b)+1)(b-a)$$

if $1 \leq a \leq 500$?

(A) 42 **(B)** 43 **(C)** 70 **(D)** 98 **(E)** 129

23. Five cubes with side lengths 1, 2, 3, 4, 5 are stacked vertically so that the centers of touching faces align. The surface area of a tower includes all exposed faces, including the bottom face. Let S be the sum of the surface areas of all $5! = 120$ possible towers. What is S?

(A) 31200 **(B)** 33600 **(C)** 34800 **(D)** 36000 **(E)** 38400

24. A fair six-sided die is rolled repeatedly. Let m_k and M_k be the minimum and maximum rolls, respectively, after k rolls of the die. Let n be the least possible n such that $M_n - m_n \geq 4$. Find the expected value of n.

(A) $\frac{7}{2}$ **(B)** 4 **(C)** 5 **(D)** $\frac{23}{4}$ **(E)** 6

25. A real number a is randomly selected from 0 to 1. What is the expected value of the number of real roots of the polynomial

$$x^9 + 3ax^6 + 3a^2x^3 - x + a^3 + a?$$

(A) $\frac{2\sqrt{3}}{3}$ **(B)** $\frac{4\sqrt{3}+9}{9}$ **(C)** $\frac{9\sqrt{3}-3}{5}$ **(D)** $\frac{5\sqrt{3}+5}{3}$ **(E)** 6